Vector autoregressive clustering for redundancy analysis in air pollution monitoring networks at Türkiye

Aytaç PEKMEZÇİ¹*, Muhammet Oğuzhan YALÇIN²

ABSTRACT
This study proposes a new approach to reduce the information redundancy at Air Pollution Monitoring Networks (APMNs) and costs required for monitoring them. Proposed approach is based on Vector Autoregressive (VAR) model which describes the relationship between multivariate time series and consists of three main steps: In the first step, VAR model between two or more than two time series consisting of air pollutant observations is estimated. This step is repeated as the number of monitoring stations (n) under study and thus, n parameter vectors are obtained. In the second step, parameters vectors are divided into homogenous groups by using clustering analysis. The objective of this step is to identify the similar monitoring stations in terms of the relationship. Last step is to calculate the reduced information redundancy and the monitoring costs. To evaluate the efficiency of proposed approach, data sets consisting of PM10 and SO2 time series obtained from 116 APMNs at Turkey are used. Fuzzy K-Medoids (FKM) as clustering method Xie-Beni (XB) index as cluster validity index are preferred. Experimental results showed that information redundancy and monitoring cost in PM10 and SO2 stations can reduced at the rate of 63.36 by following proposed approach.

Keywords: Air pollution, Information redundancy, Vector autoregressive models, Time series analysis.

INTRODUCTION
Air pollution is presence of chemicals or compounds, in the atmosphere, at levels that effect negative on human and environment health. These chemicals or compounds are generally called as “air pollutant”. Particulate Matter, Carbon Monoxide, Sulphur Dioxide (SO2), Carbon Monoxide (CO), Carbon Dioxide (CO2) and Nitrogen (N) are the most important air pollutants. Many studies have investigated the effects of the pollutants on human health and ecosystem (Ghorani -Azam et al., 2016; Kurt Kar et al., 2016; Liu et al., 2018; Landrigan et al., 2019). It is concluded that there is significant correlation between them. In order to minimize these effects, detecting of air pollution rapidly is considerably important. Air pollution monitoring network (APMN) is a main tool used for this objective. It provides an opportunity of giving correct information about air quality to public, evaluating the results of air pollution and taking precaution for protecting the environment and decreasing harmful effects of air pollution on creatures. But, APMNs require a lot of monitoring costs and need to expensive devices for monitoring. In this case, it becomes extremely important to decrease the costs required for APMNs. So far, many studies have been carried out for this objective. The most of these studies are based on detecting the stations having similar behavior in terms of an air pollutant via clustering analysis (Giri et al., 2006; Gramch et al., 2006; Lu et al., 2006; Morlini, 2007; Ignaccolo et al. 2008; Pires et al., 2008; D’Urso and Maharaj, 2009; D’Urso et al., 2015; Güler et al., 2016a, 2016b, Cotta et al., 2020). But in all of these studies, either one air pollutant is considered or analyses are carried out for each air pollutant separately and the relationship between air pollutants are not taken into account.

In this study, an approach is proposed for reducing monitoring cost in APMNs at Turkey for more than one air pollutant simultaneously. The proposed approach is based on clustering the parameters of the VAR model which indicates the relationship between air pollutants. In this way, it is aimed to get information about all air pollutants in
the model by only monitoring medoid (cluster centers) stations of air pollutant(s) selected as independent variable(s) and to decrease more the monitoring.

The organization scheme of this study can be given as follows. In Section 2, the material and methods used in this study are explained. Section 3 consists of and Section 4 concludes the study.

**MATERIAL AND METHODS**

This section briefly explains the proposed approach. In this study, the relationship between weekly PM10 and SO2 concentrations is considered. The data set are download from the website of http://laboratuvar.cevre.gov.tr/Default.ltr.aspx and each of which involves the period of between January 2018 and September 2021.

**ESTIMATING VECTOR AUTOREGRESSIVE MODELS**

VAR model is a statistical model used for investigating the relationships between two or more than two time series. VAR model between two number of time series can be defined as below:

\[ y_t = \beta_0 + \sum_{i=1}^{p_1} \beta_{1i} y_{t-i} + \sum_{i=1}^{p_2} \beta_{2i} x_{t-i} + \epsilon_{1t} \]  \hspace{1cm} (1)

\[ x_t = \alpha_0 + \sum_{i=1}^{p_1} \alpha_{1i} x_{t-i} + \sum_{i=1}^{p_2} \alpha_{2i} y_{t-i} + \epsilon_{2t} \]  \hspace{1cm} (2)

Where \( y_t \) and \( x_t \) are time series relating to different variables, \( p_1 \) and \( p_2 \) are lag length and \( \epsilon_{1t} \) and \( \epsilon_{2t} \) are error terms that follow normal distribution with zero mean and \( \sigma^2 \) variance.

The estimating of VAR model consists of several steps. These steps can be given as follows.

**Step 1: Testing stationarity**

VAR model assumes that all-time series \((y_t, x_t)\) to be analyzed are stationary, i.e., statistical properties of time series such as mean, variance and covariance are all constant over time. In order to test stationary, unit root tests are used. These tests basically examine following hypothesis.

\( H_0: \) Unit root is present in time series

\( H_1: \) Unit root is not present in time series

Where hypothesis \( H_0 \) states that time series is nonstationary. In the literature, there exists many unit root tests. In this study, Augmented Dickey Fuller (ADF) (Dickey and Fuller, 1979) has been used. ADF test statistic is calculated as follows:

\[ ADF_{\text{statistic}} = \frac{\beta_{11}}{SE(\beta_{11})} \]  \hspace{1cm} (3)

Where \( SE(\beta_{11}) \) is standard error of \( \beta_{11} \).

To decide whether time series is stationary or not, absolute value of ADF test statistic is compared with critical value (Dickey and Fuller, 1979). If this value is smaller than critical value, it is decided that time series is non-stationary. In that case, first-order difference of original time series is taken in order to make time series stationary. Unit root test is applied to differenced time series again and if differenced time series is still non-stationary, its second-order difference is taken. This process is repeated until time series become stationary. The number of taken difference
indicates stationarity order of time series. For estimating VAR model, in the other words, for performing cointegration test, time series in the model must be stationary of same order.

**Step 2: Determining lag length**  
The second step of estimating VAR model is the determination of lag length that refers to the number of previous values of time series \((y_{t-1}, y_{t-2}, ..., y_{t-p}, x_{t-1}, x_{t-2}, ..., x_{t-p_2})\). In this respect, many criteria have been employed in the literature. The most known criteria are Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Criterion (HQC).

In this study, AIC and SIC criteria are preferred for selecting lag length. These criteria are calculated as follows:

\[
AIC_p = -2n[\ln(\hat{\sigma}^2)] + 2p
\]

\[
SIC_p = n\ln(\hat{\sigma}^2) + n^{-1}p\ln(n)
\]

Where \(n\) is length of time series and \(p\) is the lag length. \(\hat{\sigma}^2\) in equations is computed as bellow:

\[
\hat{\sigma}^2 = \frac{\sum_{t=1}^{n} \epsilon_t^2}{n-p-1}
\]

The working principle of lag length selection is as follows. VAR model is estimated for various lag length. The model errors (\(\epsilon_t^2\)) and information criteria are calculated for these models. Finally lag length that provides smallest information criteria is selected.

**Step 3: Cointegration test**  
Cointegration test investigates that whether a long-run relationship between the time series exists. Johansen (JH) and Engle-Granger (EG) are widely used cointegration tests. In JH test, \(\pi = \sum_{i=1}^{p_1} \beta_i - I\) and \(\tau_i = -\sum_{j=i+1}^{p_1} \beta_j\) firstly are calculated by using parameters of estimated VAR model.

The JH test includes examination of matrix \(\pi\). Let rank of \(\pi\) be \(r\). \(r\) gives the number of cointegrated time series. In here, three possible cases arise:

1. \(r = n\) (the number of variables in the model): Time series are stationary at level.
2. \(r = 0\): There are no cointegration between time series.
3. \(r < n\): There exists \(r\) cointegrated time series.

With aim of detecting the number of cointegrated time series or whether cointegration exists or not, JH test uses two likelihood ratios known as trace test and maximum eigenvalue statistics. These statistics are calculated as follows:

\[
\lambda_{trace}(r) = -n \sum_{i=r+1}^{p_1} \ln(1 - \lambda_i)
\]

\[
\lambda_{max}(r) = -n\ln(1 - \lambda_{r+1})
\]

Where \(\lambda_i\) is the estimate value of characteristics roots of the \(\pi\) matrix. These test statistics are compared to critical values tabulated by Osterwald-Lenum (1992). If the test statistics are larger than critical value, it is decided that cointegration exists. Besides, value of \(r\) gives the number of cointegrated time series.

**Step 4: Granger causality test**  
Although cointegration indicates that time series have the long-run relationship, it does not give information about the direction of this relationship. Granger Causality (GC) test (Granger, 1969) is used for this kind of analysis. According to GC test, if previous value of \(X_t\) is useful in forecasting \(Y_t\), the \(X_t\) series is Granger causes of \(Y_t\) or if previous values of \(Y_t\) is useful in forecasting \(X_t\), the \(Y_t\) series is Granger causes of \(X_t\). Following hypothesis are testing by using GC test:

\(H_0: \beta_{21} = \beta_{22} = ... = \beta_{2p_2} = 0\) (\(X_t\) series is not Granger causes of \(Y_t\))

\(H_1: at least one of the \beta_2 coefficients \neq 0\) (\(X_t\) series is Granger causes of \(Y_t\))

\(H_0: \alpha_{21} = \alpha_{22} = ... = \alpha_{2p_2} = 0\) (\(Y_t\) series is not Granger causes of \(X_t\))
In order to test above hypothesis, F test should be calculated:

\[
F = \frac{(\text{RSS}_R - \text{RSS}_u)/p_2}{\text{RSS}_u/(T - k)}
\]

Where \(\text{RSS}_R\) is residual sum of squares relating to regression model consisting of only Y (Model 1), \(\text{RSS}_u\) is residual sum of squares for the regression model consisting of both Y variables and X variables (Model 2) and \(k\) is the number of parameters in Model 2.

The calculated F statistics is compared to critical value. If calculated F statistics is higher than critical value, it is decided that \(X_t\) series is Granger causes of \(Y_t\). Then, VAR model defined in Eq. (1) is estimated. If second hypothesis is tested and if F statistics calculated for this hypothesis is higher than critical value, it is decided that \(X_t\) series is Granger causes of \(Y_t\). Then, VAR model given in Eq. (2) is estimated.

**Step 5: Estimating VAR model**

VAR model is estimated by using Ordinary Least Squares (OLS) technique. If it is assumed that VAR model given in Eq. (1) is estimated, following equation is used:

\[
\beta = (Z'Z)^{-1}Z'Y
\]

Where \(\beta\) is parameter vector of VAR model. Matrix \(Z\) is defined as below:

\[
Z = [Y_{t-1} \cdots Y_{t-p_1} X_{t-1} \cdots X_{t-p_2}]
\]

After estimating the parameter vector \(\beta\), predicted values \((\hat{Y})\) are calculated by substituting values of \(\beta\) in Eq. (1).

**Xie-Beni (XB) INDEX**

Before applying clustering algorithm, optimal number of clusters should be determined. In this study, cluster validity indices proposed by XB (Xie and Beni, 1991) is preferred. XB index is based on two clustering criteria called as the compactness and separation. Let \(\beta = \{\beta_{11}, \beta_{12}, \ldots, \beta_{1p_1}, \beta_{21}, \beta_{22}, \ldots, \beta_{2p_2}\}\) be data set, where \(\beta_s\) is parameter vector of VAR model, \(\beta_{ij}\) s, \(i=1,2,\ldots,c, j=1,2,\ldots,p_1+p_2\) are j. component of i. cluster, \(u_{ki}, s\) \(k=1,2,\ldots,n, i=1,2,\ldots,c\) are fuzzy membership degree of k. monitoring station to i. cluster. The compactness and separation are calculated for XB index.

**COMPACTNESS:**

\[
C = \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ki} ||\beta_{ki} - \hat{\beta}_i||^2
\]

Where \(n\) is number of monitoring stations and \(c\) is the number of cluster and \(\|\|\) is Euclidian distance.

**SEPARATION:**

\[
S = \min_{i \neq j} ||\beta_i - \beta_j||^2
\]

Based on compactness and separation criteria, XB index is given as follows:

\[
XB = \frac{C}{S}
\]

XB index is calculated for all cluster numbers until predefined maximum number of clusters is reached and then the number of clusters, providing minimum XB index is set as the optimal number of clusters.

**CLUSTERING AND FUZZY K-MEDOIDS ALGORITHM**

Clustering analysis is a data mining technique used for dividing data set into groups such that data points within the same group are as similar as possible, whereas data points from different groups are as dissimilar as possible. These groups are called as cluster. Many clustering algorithms exist in the literature. This study uses FKM (Joshi and
Krishnapuram, 1999) clustering algorithm based on fuzzy clustering. In fact, the objective function in fuzzy clustering can be defined as follows:

\[ J(U, \beta_k, \bar{\beta}) = \sum_{k=1}^{c} \sum_{i=1}^{n} u_{ij}^m \| \beta_k - \bar{\beta}_i \|^2 \]  

(15)

In FKM, cluster centers are called as the medoid (\(\bar{\beta}_i\)) which corresponds to data point of a cluster whose sum of distance to all the data points in the cluster makes minimal. The reason of choosing the FKM is to find a data point in the data set as cluster center. In FKM, medoid is calculated by the following equation:

\[ \bar{\beta}_i = \text{argmin}_{1 \leq z \leq n} \sum_{t=1}^{n} u_{it}^m \| \beta_z - \beta_t \|^2 \]  

(16)

The update equation for membership degree \((u_{ij})\) is obtained as follows:

\[ u_{ij} = \frac{1}{\sum_{k=1}^{c} \left( \frac{\| \beta_i - \bar{\beta}_k \|}{\| \beta_i - \bar{\beta}_j \|} \right)^{-\frac{2}{m-1}}} \]  

(17)

The steps of FKM are as in Table 1.

<table>
<thead>
<tr>
<th><strong>Table 1. FKM algorithm.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Entering initial values</td>
</tr>
<tr>
<td>Number of clusters (c), initial medoids ((\bar{\beta}_i, i=1,2,...,c)), fuzziness index (m), termination criteria ((\epsilon)), iteration number iter = 1</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Calculating membership degrees ((u_{ij}, i=1,2,...,c, k=1,2,...,n)) by using Eq. (17)</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Increase iter by one iter = iter+1</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Calculating new values of medoids ((\bar{\beta}_i)) by using Eq. (16)</td>
</tr>
<tr>
<td><strong>Step 5:</strong> If (| \bar{\beta}^t - \bar{\beta}^{t-1} | &lt; \epsilon) iteration is terminated, otherwise go to Step 2.</td>
</tr>
<tr>
<td><strong>Step 6:</strong> Assign data points ((\beta_k)) to clusters according to maximum membership degrees.</td>
</tr>
</tbody>
</table>

**REDUNDANCY ANALYSIS**

For redundancy analysis, below steps are followed.

- Stationarity test is separately performed for each of all PM\(_{10}\) and SO\(_2\) time series. The first difference of non-stationary PM\(_{10}\) and SO\(_2\) series are taken. Unit root test is re-applied and if the differenced time series is still non-stationary, its second-order difference is taken. This process is repeated until PM\(_{10}\) and SO\(_2\) series become stationary. The number of taken difference indicates stationarity order of the time series. In order to estimate VAR model, PM\(_{10}\) and SO\(_2\) series obtained from the same station must be stationary of same order. The next steps of the analysis are continued with same-order stationary stations. PM\(_{10}\) and SO\(_2\) stations which are not same-order stationary are continued to be monitored. When the number of these stations is considered as \(n_1\), the number of stations which are continued to be monitored is determined as \(2n_1\) \((n_1\) number of PM\(_{10}\) and SO\(_2\)) in this step.
- In the second step, PM\(_{10}\) and SO\(_2\) time series having long-time relationship are determined by using Cointegration test. If the number of non-cointegrated series is equal to \(n_2\), 2\(n_2\) stations are continued to monitor at this step.
- Dependent and independent variables are determined by using GC test. 2\(n_3\) stations with no causality relationship are continued to monitor. The parameters of VAR model are estimated for the stations remained.
- The number of cluster \(c\) is determined by using these parameters and FKM clustering algorithm is applied. Lastly, below equation is used to calculate the percentage of the decreased monitoring cost (PDR-MC).

\[ PDR - MC = \left( 1 - \frac{(2xn_1 + 2xn_2 + 2xn_3 + c)}{2xN} \right) \times 100 \]  

(18)
Where $N$ is total number of monitoring stations, $c$ is the number medoid stations to be monitored from air pollutant determined as independent variable.

RESULTS AND DISCUSSION

To identify stations that do not require to be monitored and thus reduce the monitoring cost, the procedure given in Fig1. is followed.

**Fig. 1.** The procedure followed in this study.

When the procedure given in Fig1. is followed, below results are found.

- According to ADF test, it is found that all PM$_{10}$ and SO$_2$ time series are stationary of same order. Thus, the value of $n_1$ is determined as zero and no monitoring station has been eliminated at this step.
- The lag lengths are selected between 2 and 12.
- Since all time series are stationary at the level, no station is removed from the analysis ($(n_2 = 0)$).
- GC test arise that PM$_{10}$ series are Granger cause of SO$_2$ series for 75 stations. According to this result, SO$_2$ series are selected as dependent variables and PM$_{10}$ series are selected as independent variables in the VAR model. This means that PM$_{10}$ concentrations can be used to estimate SO$_2$ concentrations.

Besides, 41 PM$_{10}$ and SO$_2$ stations with no causality relationship are removed from the analysis and thus it is concluded that these stations are required to monitor. Table 2 shows these stations.
Table 2. The stations which continue to be monitored.

Name of Stations

<table>
<thead>
<tr>
<th>Stations</th>
<th>β₀</th>
<th>β₁</th>
<th>Stations</th>
<th>β₀</th>
<th>β₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adana</td>
<td>0.0000</td>
<td>0.1349</td>
<td>İzmir</td>
<td>7.8732</td>
<td>0.1731</td>
</tr>
<tr>
<td>Doğankent</td>
<td>8.9639</td>
<td>0.1267</td>
<td>Gaziemir</td>
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<td>0.0069</td>
</tr>
<tr>
<td>Valilik</td>
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<td>0.1957</td>
<td>Şirinyer IBB</td>
<td>12.4022</td>
<td>-0.0429</td>
</tr>
<tr>
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<td>0.2966</td>
<td>Kahramanmaras Elbistan</td>
<td>5.7135</td>
<td>0.1110</td>
</tr>
<tr>
<td>Ağırı Doğubeyazıt</td>
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<td>0.1748</td>
<td>Karabük Kardemir 1</td>
<td>18.0263</td>
<td>0.0913</td>
</tr>
<tr>
<td>Patnos</td>
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<td>0.3371</td>
<td>Karabük Tören Alanı</td>
<td>6.2590</td>
<td>0.3190</td>
</tr>
<tr>
<td>Amasya</td>
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<td>0.0549</td>
<td>Karaman</td>
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<td>0.0901</td>
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<tr>
<td>Merzifon</td>
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<td>Kars</td>
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</tr>
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<td>0.3811</td>
</tr>
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<td>Sinean</td>
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<td>0.1016</td>
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<tr>
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<td>Konya Meram</td>
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<td>Malatya</td>
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</tr>
</tbody>
</table>

According to this, analyses are continued with 75 number of PM\(_{10}\) and SO\(_2\) stations. VAR model denoted the relationship between SO\(_2\) and PM\(_{10}\) is estimated for each of these stations. Table 3 shows the parameters of these models.

Table 3. The parameters of VAR models.

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<td>0.0236</td>
<td>Konya Meram</td>
<td>5.1902</td>
<td>0.1862</td>
</tr>
<tr>
<td>Bayburt</td>
<td>5.4535</td>
<td>0.0409</td>
<td>Malatya</td>
<td>9.0593</td>
<td>0.0343</td>
</tr>
</tbody>
</table>
Table 4 provides the results of XB index.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>XB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.98</td>
</tr>
<tr>
<td>3</td>
<td><strong>3.67</strong></td>
</tr>
<tr>
<td>4</td>
<td>4.83</td>
</tr>
<tr>
<td>5</td>
<td>8.54</td>
</tr>
<tr>
<td>6</td>
<td>15.38</td>
</tr>
<tr>
<td>7</td>
<td>177626.7</td>
</tr>
<tr>
<td>8</td>
<td>155900.7</td>
</tr>
<tr>
<td>9</td>
<td>138645.6</td>
</tr>
<tr>
<td>10</td>
<td>123923.8</td>
</tr>
</tbody>
</table>

According to Table 4, the optimal number of clusters is found as 3 since it has the smallest XB index. This states that there are three different groups in terms of the relationship between PM<sub>10</sub> and SO<sub>2</sub> in Turkey. When FKM clustering...
algorithm with 3 number of clusters is applied to the parameters given in Table 3, the clusters given in Table 5 are constituted.

Table 5. Stations for each cluster and medoid stations.

<table>
<thead>
<tr>
<th>Cluster Number</th>
<th>Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adana Doğankent, Amasya, Amasya Merzifon, Ankara Sihhiye, Ankara Sincan, Ankara Siteler, Bartın, Batman, Bayburt, Bursa İneggöl, Çanakkale, Diyarbakır, Iğdır Aralık, İstanbul Kandilli, İzmir Çiğli IBB, İzmir Gaziemir, İzmir Şirinyer, Kahramanmaraş Elbistan, Karabük Kardemir 1, Karabük Tören Alanı, Karaman, Kırşehir, Kocaeli Gebze, Konya Karkent Sanayi, Konya Meram, Malatya, Muğla, Ordu Ünye, Siirt, Sinop Boyabat, Sivas Meteroloji</td>
</tr>
<tr>
<td>3</td>
<td>Edirne Keşan, Hakkari, Tekirdağ Merkez</td>
</tr>
</tbody>
</table>

The results obtained from Table 5 can be interpreted as follows:

- Clusters consist of 31, 41 and 3 stations respectively.
- When the results are interpreted according to cluster 3, it can be said that the stations Edirne Keşan, Hakkari and Tekirdağ Merkez have similar behavior in terms of the relationship between PM$_{10}$ and SO$_2$. SO$_2$ and PM$_{10}$ values of all these stations can be estimated by only monitoring Tekirdağ Merkez PM$_{10}$ station. It is possible to interpret the other clusters similarly.
- Medoids are Kahramanmaraş Elbistan, Samsun Canik and Tekirdağ Merkez respectively and PM$_{10}$ stations relating to the medoids should be continued to monitor.

From this, the percentage of the decreased monitoring cost can be determined as follows.

$$PDR - MC = \left(1 - \frac{(2x0 + 2x0 + 2x41 + 3)}{2x116}\right) \times 100 = 63.36\%$$

As a consequence of analyses, the results obtained can be summarized as follows:

- The analyses are started with 116 number of PM$_{10}$ and SO$_2$ monitoring stations at Turkey.
- In the first step of building VAR models, it is observed that all pairs of PM$_{10}$ and SO$_2$ are stationary of same order. Thus, no monitoring station is eliminated in this step.
- In the second step of building VAR models, it is concluded that there exists long-term relationship between all pairs of PM$_{10}$ and SO$_2$.
- According to the results of Granger Causality test, causality relationship between 41 pairs of PM$_{10}$ and SO$_2$ are not found. It is decided that the monitoring should be continued for these stations. These stations are determined as Adana_Meteroloji, Aksaray, Amasya_Suluova, Ankara_Kayaş, Ankara_Keçiören, Ardahan,

- Granger Causality test revealed that PM$_{10}$ concentrations are Granger cause of SO$_2$ concentrations. Therefore, VAR models are estimated such that dependent variables are SO$_2$ and independent variables PM$_{10}$.
- Xie-Beni index found that optimal number of clusters is equal to three. This means that there exist three groups which have different behavior in terms of the relationship between PM$_{10}$ and SO$_2$ concentrations at Turkey.
- The parameters of VAR models estimated for 75 monitoring stations are clustered by using FKM algorithm. As a result of clustering, the stations that represent the clusters are found as Kahramanmaraş Elbistan, Samsun Canik, Tekirdağ Merkez.
- The number of stations to be monitored is found as 85 (41 PM$_{10}$ +3 PM$_{10}$+41 SO$_2$). Thus, it is concluded that the monitoring cost and information redundancy at Turkey are decreased at rate of 63.36% by only monitoring 85 of 232 stations. This reduction in air monitoring cost means that the total cost (manpower, money, time, etc.) required for the future prediction values of PM10 and SO2 variables at each station is reduced.

So far, many studies have been carried out with aim of optimizing the number of APMNs and reducing the monitoring cost. But, in all of these studies, either only one air pollutant is considered, or analyses are carried out for each air pollutant separately. There is no study that takes into the relationship between air pollutants. This study proposes an approach based on the relationship between air pollutants for decreasing monitoring cost. Thus, it is aimed to get information about the other air pollutants by monitoring medoid stations relating to only one air pollutant and aimed to reduce monitoring cost more. In this study, relationship between SO$_2$ and PM$_{10}$ air pollutants are taken into account. It is possible that redundancy analysis approach proposed in this study is carried out for the other pollutants and the other regions.

The originality of this study is that while most of the studies in the literature were on a single variable, this study used more than one variable (PM10 and SO2). The contribution of this study to the literature is to propose a new approach based on the relationship between multiple air pollutants to reduce the information redundancy and the cost of monitoring at air pollution monitoring stations.

In future studies, it can be compared using other clustering algorithms other than Fuzzy K-Medoids clustering algorithm.

REFERENCES


